

**DELHI UNIV. B.COM.(HONS.)/ IV SEM./2018
PAPER BCH 4.2 - BUSINESS MATHEMATICS**

Time : 3 hours

Maximum Marks : 75

Attempt All questions. Marks are indicated against each question.

1. (a) One unit of commodity A is produced by combining 1 unit of land, 2 units of labour and 5 units of capital. One unit of B is produced by combining 2 units of land, 3 units of labour and 1 unit of capital, one unit of C is produced by 3 units of land, 1 unit of labour and 2 units of capital. If the price of A, B and C are ₹ 27, ₹ 16 and ₹ 19 respectively, find the rent R, wages W and rate of interest I. 4

Or

The following matrix gives the proportionate mix of constituents used for the fertilisers:

	A	B	C	D
I	0.5	0	0.5	0
Fertilizers II	0.2	0.3	0	0.5
III	0.2	0.2	0.1	0.5

- (i) If sales are 1000 tins (of one kilogram) per week, 20% being fertilizer 1, 30% being fertilizer 2 and 50% being fertilizer 3, how much of each constituent is used?
- (ii) If the cost of each constituent is 50 paise, 60 paise, 75 paise and 100 paise per 100 grams, respectively, how much does a one kilogram tin of each fertilizer cost?
- (iii) What is the total cost per week? 4
- (b) A hypothetical economy produces only two commodities X and Y. The two commodities serve as intermediate inputs in each other's production. To produce a unit of X, 0.2 unit of X and 0.6 unit of Y are needed. To produce a unit of Y, 0.4 unit of X and 0.3 unit of Y are needed. 3 and 5 labour hours are required to produce a unit of X and a unit of Y respectively. The wage rate is ₹ 20 per labour hour. If the final demand of X increases by 150 units and that of Y decreases by 120 units, find:
- (i) Change in the gross outputs of each of the two commodities,
- (ii) Change in labour requirement,
- (iii) Change in the value-added in the two producing sectors. 8

Or

A three sector economy has the following input output coefficient matrix:

$$A = \begin{pmatrix} 0 & 0.5 & 0 \\ 0.2 & 0 & 0.5 \\ 0.4 & 0 & 0 \end{pmatrix}$$

The labour days required per unit of output of the three sectors are 0.4, 0.7 and 1.2 respectively and their consumer output targets are 1000, 5000 and 4000 units respectively. By using matrix algebra, find

- (i) The gross output of each sector
- (ii) Total labour days required.

2. (a) Solve the following linear programming problem by simplex method.

$$\text{Max.} \quad : \quad Z = 2x_1 + 4x_2$$

$$\text{Subject to} \quad : \quad 2x_1 + x_2 \leq 18$$

$$3x_1 + 2x_2 \geq 30$$

$$x_1 + 2x_2 = 26$$

$$x_1, x_2 \geq 0.$$

Also find the dual of the above problem.

Or

Given the following initial simplex table of minimization problem:

BV	X_1	X_2	S_1	S_2	A_1	A_2	Quantity
A_1	3	1	-1	0	1	0	27
A_2	1	1	0	0	0	1	21
S_2	1	2	0	1	0	0	40
$C \rightarrow$	3	4	0	0	M	M	

(i) Write the corresponding linear programming problem.

(ii) Solve it by simplex method.

(iii) Is it a case of multiple solutions? If yes, also find the alternative solution.

(b) A firm produces and sells two products A and B. The profit per unit of A is ₹ 40 and per unit of B ₹ 30. The products are processed on the same machine but sold in two different markets. It takes three times of machine time to produce a unit of A as compared to a unit of B, and if all time is devoted to producing A, the machine can produce a maximum of 10,000 units. The market research indicates that the firm can sell a maximum of 8,000 units of A and 15,000 units of B.

Formulate the above as a linear programming problem to maximize profit and solve the same by graphical method.

Or

Using graphical method in a linear programming problem, explain how would you identify the case of :

- (i) Redundant constraint,
- (ii) No solution,
- (iii) Unbounded solution,
- (iv) Multiple optimal solution.

Give a rough sketch of each case.

6

3. (a) Find the price elasticities of demand and supply at equilibrium point for the demand function $p^d = \sqrt{100 - x^2}$ and supply function $x^s = 2p - 10$, where p is price and x is quantity.

6

Or

The cost of producing x units of TV sets by a monopolist is given $TC = \frac{x^2}{25} + 3x + 100$ and the demand Function is given by $x = 75 - 3p$ (where p is the price). If a tax of t per set is imposed by the government, determine the monopolists output and price under the new situation. Find the value of t that maximises tax revenue to the government.

6

- (b) A publishing house purchases 2000 units of a particular item per year at a unit cost of ₹ 20. The ordering cost per order is ₹ 50 and the inventory cost is 25% of the value of the average inventory. Find the optimal order quantity and minimum total cost. If the supplier offers a discount of 3% on purchases in lots of 1000, should the offer be accepted?

6

Or

The Demand function and the total cost function of a monopolist are as follows:

$$p = 100 - 3q + 4A^{1/2}$$

$$C = 4q^2 + 10q + A$$

where A is the level of advertisement expenditure. Find the values of A , p and q to maximize profit.

6

- (c) Suppose a firm has a production function $Q = AK^\alpha L^{1-\alpha}$ where $A > 0$ and $0 < \alpha < 1$. Show that production function has diminishing returns to inputs and constant returns to scale.

6

Or

Find the MRTS and the elasticity of substitution for the following CES production function:

$$x = f(L, K) = [\alpha K^\theta + (1 - \alpha) L^\theta]^{1/\theta}$$

(where x is the total output obtained by using L and K units of labour and capital respectively, and α and θ are constants.)

4. (a) A small business is considering buying an energy saving device which will reduce its cost of fuel. The device will cost ₹ 32,000. Savings in rupees will occur at the rate of $S(t) = 20,000e^{-0.1t}$, where t equals time measured in years. Determine how long will it take for the firm to recover the cost of the device. 6

Or

A monopolist's demand function is $x = 240 - 10p$, where x is quantity demanded and p is price per unit. With the average cost function $AC = 10 + \frac{x}{25}$, find the consumer's surplus at the price which monopolist will like to fix to maximise profit. 6

(b) Suppose that when it is t years old, an industrial machine generates revenue at the rate of $R(t) = 6025 - 8t^2$ rupees per year and results in costs that accumulate at the rate of $C(t) = 4681 + 13t^2$ rupees per year:

(i) For how many years is the use of the machine profitable?

(ii) What are the net earnings generated by the machine during its period of profitability? 6

Or

Given the marginal revenue function $MR = \frac{4}{(2x+3)^2} - 1$, show that average revenue

function is $P = \frac{4}{6x+9} - 1$. 6

5. (a) Mr. X deposited ₹ 50,000 in a bank for five years offering interest at the rate of 8% per annum compounded half yearly during first two years, at the rate of 10% per annum compounded quarterly for the third year and at 9% per annum compounded continuously for the next two years. Find his balance after five years. 5

Or

A debt of ₹ 3,000 which is due 6 years from now, is instead to be paid off by three payments ₹ 500 now, ₹ 1,500 in 3 years and a final payment of ₹ 475 at the end of n years. If the rate of interest is 6% per annum effective, find the value of n . 5

(b) A loan of ₹ 10,000 is to be repaid by equal annual instalments of principal and interest over a period of 20 years. The rate of interest is 3% per annum effective. Find:

(i) The annual installment;

(ii) The capital contained in 8th installment; and

(iii) The principal repaid after 12 installments have been paid.

Or

A machine bought for ₹ 50,000 is depreciated at 10% p.a. on written down basis for 8 years. To make up the loss due to depreciation a sinking fund is created by setting aside a sum of money every year. What is the value of each payment if the accumulated amount in the fund is equal to total depreciation and if the interest is 5% p.a. compounded annually?

5

- (c) An orchard will yield its first full crop at the end of 5 years and is expected to maintain an annual income of ₹ 5,000 for 20 years in all. Find the cash price of the orchard if money is worth 3% per annum.

5

Or

Machine A costs ₹ 25,000 and has a useful life of 8 years. Machine B costs ₹ 28,000 and has a useful life of 6 years. Suppose Machine A generates an annual savings of ₹ 5,000 while Machine B generates an annual savings of ₹ 5,500. Assuming the time value of money is 7% effective, which machine is preferable?

5